

Effects of quantum deformation on the spin-1/2 Aharonov-Bohm problem

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Abstract

In this letter we study the Aharonov-Bohm problem for a spin-1/2 particle in the quantum deformed framework generated by the κ -Poincaré-Hopf algebra. We consider the nonrelativistic limit of the κ -deformed Dirac equation and use the spin-dependent term to impose an upper bound on the magnitude of the deformation parameter ϵ . By using the self-adjoint extension approach, we examine the scattering and bound state scenarios. After obtaining the scattering phase shift and the S -matrix, the bound states energies are obtained by analyzing the pole structure of the latter. Using a recently developed general regularization prescription [Phys. Rev. D. **85**, 041701(R) (2012)], the self-adjoint extension parameter is determined in terms of the physics of the problem. For last, we analyze the problem of helicity conservation.

Keywords: κ -Poincaré-Hopf algebra, self-adjoint extension, Aharonov-Bohm, scattering, helicity

1. Introduction

Theory of quantum deformations based on the κ -Poincaré-Hopf algebra has been a alternative framework for studying relativistic and nonrelativistic quantum systems. The Hopf-algebraic description of κ -deformed Poincaré symmetries, with κ a masslike fundamental deformation parameter, was introduced in [1, 2]. In this context, the space-like κ -deformed Minkowski spacetime is the more interesting among them because its phenomenological applications. Such κ -deformed Poincaré-Hopf algebra established in Refs. [1–6] is defined by the following commutation relations

$$[\Pi_\nu, \Pi_\mu] = 0, \quad (1a)$$

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$$[M_i, \Pi_\mu] = (1 - \delta_{0\mu}) i\epsilon_{ijk} \Pi_k, \quad (1b)$$

$$[L_i, \Pi_\mu] = i[\Pi_i]^{\delta_{0\mu}} [\delta_{ij} \varepsilon^{-1} \sinh(\varepsilon \Pi_0)]^{1-\delta_{0\mu}}, \quad (1c)$$

$$[M_i, M_j] = i\epsilon_{ijk} M_k, \quad [M_i, L_j] = i\epsilon_{ijk} L_k, \quad (1d)$$

$$[L_i, L_j] = -i\epsilon_{ijk} \left[M_k \cosh(\varepsilon \Pi_0) - \frac{\varepsilon^2}{4} \Pi_k \Pi_l M_l \right], \quad (1e)$$

where ε is defined by

$$\varepsilon = \kappa^{-1} = \lim_{R \rightarrow \infty} (R \ln q), \quad (2)$$

with R being the de Sitter curvature and q is a real deformation parameter, $\Pi_\mu = (\Pi_0, \mathbf{\Pi})$ are the κ -deformed generators for energy and momenta. Also, the M_i , L_i represent the spatial rotations and deformed boosts generators, respectively. The coalgebra and antipode for the κ -deformed Poincaré algebra was established in Ref. [7].

The physical properties of κ -deformed relativistic quantum systems can be accessed by solving the κ -deformed Dirac equation [3, 4, 8, 9]. The deformation parameter κ can be usually interpreted as being the Planck mass M_P [10]. The κ -deformation has implications for various properties of physical systems as for example, vacuum energy divergent [11], Landau levels [12], spin-1/2 Aharonov-Bohm (AB) interaction creating additional bound states [13], Dirac oscillator [14], Dirac-Coulomb problem [4] and constant magnetic interaction [15]. In Ref. [13] the spin-1/2 AB problem was solved for the first time in connection with the theory of quantum deformations. The AB problem [16] has been extensively studied in different contexts in recent years [17–24]. In this letter we study the scattering scenario of the model addressed in Ref. [13] where only the bound state problem was considered. We solve the problem by following the self-adjoint extension approach [25–27] and by using the general regularization prescription proposed in [20] we determine the self-adjoint extension parameter in terms of the physics of the problem. Such procedure allows discuss the problem of helicity conservation and, as a alternative approach, we obtain the bound states energy from the poles of S -matrix.

The plan of our Letter is the following. In Section 2 we introduce the κ -deformed Dirac equation to be solved and take its nonrelativistic limit in order to study the physical implications of κ -deformation in the spin-1/2 AB problem. A new contribution to the nonrelativistic Hamiltonian arises in this approach. These new term imply a direct correction on the anomalous magnetic moment term. We impose a upper bound on the magnitude of the deformation parameter ε . The Section 3 is devoted to study the κ -deformed Hamiltonian via self-adjoint extension approach and presented some important properties of the κ -deformed wave function. In Section 4 are addressed the scattering and bound states scenario within the framework of κ -deformed Schrödinger-Pauli equation. Expressions for the phase shift, S -matrix, and bound states are derived. We also derive a relation between the self-adjoint extension parameter and the physical parameters of the problem. For last, we make a detailed analysis of the helicity conservation problem in the present framework. A brief conclusion is outlined in Section 5.

2. κ -deformed Schrödinger-Pauli equation

In the minimal coupling prescription the (3+1)-dimensional κ -deformed Dirac equation supported by the algebra in Eq. (1) up to $O(\varepsilon)$ order was derived in Ref. [13] (see also Refs. therein). We here analyze the (2+1)-dimensional κ -deformed Dirac equation, which follows from the decoupling of (3+1)-dimensional κ -deformed Dirac equation for the specialized case where $\partial_3 = 0$ and $A_3 = 0$, into two uncoupled two-component equations, such as implemented in Refs. [28–30]. This way, the planar κ -deformed Dirac equation ($\hbar = c = 1$) is

$$\hat{H}\psi = \left[\beta\boldsymbol{\gamma} \cdot \boldsymbol{\Pi} + \beta M + \frac{\varepsilon}{2} (M\boldsymbol{\gamma} \cdot \boldsymbol{\Pi} + es\boldsymbol{\sigma} \cdot \mathbf{B}) \right] \psi = \bar{E}\psi, \quad (3)$$

where ψ is a two-component spinor, $\boldsymbol{\Pi} = \mathbf{p} - e\mathbf{A}$ is the generalized momentum, and s is twice the spin value, with $s = +1$ for spin “up” and $s = -1$ for spin “down”. The γ -matrices in (2 + 1) are given in terms of the Pauli matrices

$$\beta = \gamma_0 = \sigma_3, \quad \gamma_1 = i\sigma_2, \quad \gamma_2 = -is\sigma_1. \quad (4)$$

Here few comments are in order. First, the κ -deformed Dirac equation is defined in the commutative spacetime and the corresponding γ -matrices are independent of the deformation parameter κ [31]. Second, it is important to observe that in Ref. [13] the authors only consider the negative value of the spin projection, here our approach considers a more general situation.

We shall now take the nonrelativistic limit of Eq. (3). Writing $\psi = (\chi, \phi)^T$, where χ and ϕ are the “large” and “small” components of the spinor, and using $\bar{E} = M + E$ with $M \gg E$, after expressing the lower component ϕ in terms of the upper one, χ , we get the κ -deformed Schrödinger-Pauli equation for the large component

$$H\chi = E\chi, \quad (5)$$

with

$$H = \frac{1}{2M} [\Pi_1^2 + \Pi_2^2 - (1 - M\varepsilon)esB_3], \quad (6)$$

where it was assumed that $\varepsilon^2 \cong 0$. It can be seen from (6) that the magnetic moment has modified by a quantity proportional to the deformation parameter.

Another effect enclosed in Hamiltonian (6) is concerned with the anomalous magnetic moment of the electron. The electron magnetic moment is $\boldsymbol{\mu} = -\boldsymbol{\mu}\boldsymbol{\sigma}$, with $\mu = e/2M$, and $g = 2$ the gyromagnetic factor. The anomalous magnetic moment of the electron is given by $g = 2(1 + a)$, with $a = \alpha/2\pi = 0.00115965218279$ representing the deviation in relation to the usual case [32]. In this case, the magnetic interaction is $\bar{H} = \mu(1 + a)(\boldsymbol{\sigma} \cdot \mathbf{B})$. In accordance with very precise measurements and quantum electrodynamics (QED) calculations [33], precision corrections to this factor are now evaluated at the level of 1 part in 10^{11} , that is, $\Delta a \leq 3 \times 10^{-11}$. In our case, the Hamiltonian (6) provides κ -tree-level contributions to the usual $g = 2$ gyromagnetic factor, which can not be larger than $a = 0.00116$ (the current experimental value for the anomalous magnetic moment). The total κ -deformed magnetic interaction in Eq. (6) is

$$H_{\text{magn}} = (1 - M\varepsilon)s(\boldsymbol{\mu} \cdot \mathbf{B}). \quad (7)$$

For the magnetic field along the z -axis and a spin-polarized configuration in the z -axis, this interaction assumes the form

$$(1 - M\varepsilon) s\mu B_z, \quad (8)$$

with $M\varepsilon$ representing the κ -tree-level correction that should be smaller than 0.00116. Under such consideration, we obtain the following upper bound for ε :

$$\varepsilon < 2.27 \times 10^{-9} (eV)^{-1}, \quad (9)$$

where we have used $M = 5.11 \times 10^5 eV$.

We now pass to study the κ -deformed Schrödinger-Pauli equation in the AB background potential [16]. The vector potential of the AB interaction, in the Coulomb gauge, is

$$\mathbf{A} = -\frac{\alpha}{r} \hat{\boldsymbol{\varphi}}, \quad A_0 = 0, \quad (10)$$

where $\alpha = \Phi/\Phi_0$ is the flux parameter with $\Phi_0 = 2\pi/e$. The magnetic field is given in the usual way

$$e\mathbf{B} = e\nabla \times \mathbf{A} = -\alpha \frac{\delta(r)}{r} \hat{\mathbf{z}}. \quad (11)$$

So, the κ -deformed Schrödinger-Pauli equation can be written as

$$\frac{1}{2M} \left[H_0 + \eta \frac{\delta(r)}{r} \right] \chi = E\chi, \quad (12)$$

with

$$H_0 = \left(\frac{1}{i} \nabla - e\mathbf{A} \right)^2, \quad (13)$$

and

$$\eta = (1 - M\varepsilon)\alpha s, \quad (14)$$

is the coupling constant of the $\delta(r)/r$ potential.

For the present system the total angular momentum operator in the z direction,

$$\hat{J}_3 = -i\partial_\varphi + \frac{1}{2}\sigma_3, \quad (15)$$

commutes with the effective Hamiltonian. So, it is possible to express the eigenfunctions of the two dimensional Hamiltonian in terms of the eigenfunctions of \hat{J}_3 . The eigenfunctions of this operator are

$$\psi = \begin{pmatrix} \chi \\ \phi \end{pmatrix} = \begin{pmatrix} f_m(r) e^{i(m_j-1/2)\varphi} \\ g_m(r) e^{i(m_j+1/2)\varphi} \end{pmatrix}, \quad (16)$$

with $m_j = m + 1/2 = \pm 1/2, \pm 3/2, \dots$, with $m \in \mathbb{Z}$. Inserting this into Eq. (12), we can extract the radial equation for $f_m(r)$ ($k^2 = 2ME$)

$$hf_m(r) = k^2 f_m(r), \quad (17)$$

where

$$h = h_0 + \eta \frac{\delta(r)}{r}, \quad (18)$$

$$h_0 = -\frac{d^2}{dr^2} - \frac{1}{r} \frac{d}{dr} + \frac{(m + \alpha)^2}{r^2}. \quad (19)$$

The Hamiltonian in Eq. (18) is singular at the origin. This problem can then be treated by the method of the self-adjoint extension [27], which we pass to discuss in the next Section.

3. Self-adjoint extension analysis

The operator h_0 , with domain $\mathcal{D}(h_0)$, is self-adjoint if $h_0^\dagger = h_0$ and $\mathcal{D}(h_0^\dagger) = \mathcal{D}(h_0)$. For smooth functions, $\xi \in C_0^\infty(\mathbb{R}^2)$ with $\xi(0) = 0$, we should have $h\xi = h_0\xi$, and hence it is reasonable to interpret the Hamiltonian (18) as a self-adjoint extension of $h_0|_{C_0^\infty(\mathbb{R}^2/\{0\})}$ [34–36]. In order to proceed to the self-adjoint extensions of (19), we decompose the Hilbert space $\mathfrak{H} = L^2(\mathbb{R}^2)$ with respect to the angular momentum $\mathfrak{H} = \mathfrak{H}_r \otimes \mathfrak{H}_\varphi$, where $\mathfrak{H}_r = L^2(\mathbb{R}^+, r dr)$ and $\mathfrak{H}_\varphi = L^2(\mathcal{S}^1, d\varphi)$, with \mathcal{S}^1 denoting the unit sphere in \mathbb{R}^2 . The operator $-\partial^2/\partial\varphi^2$ is essentially self-adjoint in $L^2(\mathcal{S}^1, d\varphi)$ [37] and we obtain the operator h_0 in each angular momentum sector. Now, using the unitary operator $U : L^2(\mathbb{R}^+, r dr) \rightarrow L^2(\mathbb{R}^+, dr)$, given by $(U\xi)(r) = r^{1/2}\xi(r)$, the operator h_0 becomes

$$\tilde{h}_0 = Uh_0U^{-1} = -\frac{d^2}{dr^2} - \left[(m + \alpha)^2 - \frac{1}{4} \right] \frac{1}{r^2}, \quad (20)$$

which is essentially self-adjoint for $|m + \alpha| \geq 1$, while for $|m + \alpha| < 1$ it admits a one-parameter family of self-adjoint extensions [37], h_{0,λ_m} , where λ_m is the self-adjoint extension parameter. To characterize this family we will use the approach in [26], which is based in a boundary conditions at the origin.

Following the approach in Refs. [26, 27], all the self-adjoint extensions h_{0,λ_m} of h_0 are parametrized by the boundary condition at the origin

$$f_{0,\lambda_m} = \lambda_m f_{1,\lambda_m}, \quad (21)$$

with

$$f_{0,\lambda_m} = \lim_{r \rightarrow 0^+} r^{|m+\alpha|} f_m(r), \quad (22)$$

$$f_{1,\lambda_m} = \lim_{r \rightarrow 0^+} \frac{1}{r^{|m+\alpha|}} \left[f_m(r) - f_{0,\lambda_m} \frac{1}{r^{|m+\alpha|}} \right], \quad (23)$$

where $\lambda_m \in \mathbb{R}$ is the self-adjoint extension parameter. The self-adjoint extension parameter λ_m has a physical interpretation, it represents the scattering length [38] of h_{0,λ_m} [27]. For $\lambda_m = 0$ we have the free Hamiltonian (without the δ function) with regular wave functions at the origin, and for $\lambda_m \neq 0$ the boundary condition in Eq. (21) permit a $r^{-|m+\alpha|}$ singularity in the wave functions at the origin.

4. Scattering and bound states analysis

The general solution for Eq. (17) in the $r \neq 0$ region can be written as

$$f_m(r) = a_m J_{|m+\alpha|}(kr) + b_m Y_{|m+\alpha|}(kr), \quad (24)$$

with a_m and b_m being constants and $J_\nu(z)$ and $Y_\nu(z)$ are the Bessel functions of first and second kind, respectively. Upon replacing $f_m(r)$ in the boundary condition (21), we obtain

$$\lambda_m a_m v k^{|m+\alpha|} = b_m \left[\zeta k^{-|m+\alpha|} - \lambda_m (\beta k^{|m+\alpha|} + \zeta \nu k^{-|m+\alpha|} \lim_{r \rightarrow 0^+} r^{2-2|m+\alpha|}) \right], \quad (25)$$

where

$$\begin{aligned} v &= \frac{1}{2^{|m+\alpha|} \Gamma(1 + |m + \alpha|)}, & \zeta &= -\frac{2^{|m+\alpha|} \Gamma(|m + \alpha|)}{\pi}, \\ \beta &= -\frac{\cos(\pi|m + \alpha|) \Gamma(-|m + \alpha|)}{\pi 2^{|m+\alpha|}}, & \nu &= \frac{k^2}{4(1 - |m + \alpha|)}. \end{aligned} \quad (26)$$

In Eq. (25), $\lim_{r \rightarrow 0^+} r^{2-2|m+\alpha|}$ is divergent if $|m + \alpha| \geq 1$, hence b_m must be zero. On the other hand, $\lim_{r \rightarrow 0^+} r^{2-2|m+\alpha|}$ is finite for $|m + \alpha| < 1$, it means that there arises the contribution of the irregular solution $Y_{|m+\alpha|}(kr)$. Here, the presence of an irregular solution contributing to the wave function stems from the fact the Hamiltonian h is not a self-adjoint operator when $|m + \alpha| < 1$ (cf., Section 3), hence such irregular solution must be associated with a self-adjoint extension of the operator h_0 [39, 40]. Thus, for $|m + \alpha| < 1$, we have

$$\lambda_m a_m v k^{|m+\alpha|} = b_m (\zeta k^{-|m+\alpha|} - \lambda_m \beta k^{|m+\alpha|}), \quad (27)$$

and by substituting the values of v , ζ and β into above expression we find

$$b_m = -\mu_m^{\lambda_m} a_m, \quad (28)$$

where

$$\mu_m^{\lambda_m} = \frac{\lambda_m k^{2|m+\alpha|} \Gamma(1 - |m + \alpha|) \sin(\pi|m + \alpha|)}{B_k}, \quad (29)$$

and

$$\begin{aligned} B_k &= \lambda_m k^{2|m+\alpha|} \Gamma(1 - |m + \alpha|) \cos(\pi|m + \alpha|) \\ &\quad + 4^{|m+\alpha|} \Gamma(1 + |m + \alpha|). \end{aligned} \quad (30)$$

Since a δ function is a very short range potential, it follows that the asymptotic behavior of $f_m(r)$ for $r \rightarrow \infty$ is given by [41]

$$f_m(r) \sim \sqrt{\frac{2}{\pi k r}} \cos \left(kr - \frac{|m|\pi}{2} - \frac{\pi}{4} + \delta_m^{\lambda_m}(k, \alpha) \right), \quad (31)$$

where $\delta_m^{\lambda_m}(k, \alpha)$ is a scattering phase shift. The phase shift is a measure of the argument difference to the asymptotic behavior of the solution $J_{|m|}(kr)$ of the radial free equation which is regular at the origin. By using the asymptotic behavior of the Bessel functions [42] into Eq. (24) we obtain

$$f_m(r) \sim a_m \sqrt{\frac{2}{\pi kr}} \left[\cos \left(kr - \frac{\pi|m + \alpha|}{2} - \frac{\pi}{4} \right) - \mu_m^{\lambda_m} \sin \left(kr - \frac{\pi|m + \alpha|}{2} - \frac{\pi}{4} \right) \right]. \quad (32)$$

By comparing the above expression with Eq. (31), we found

$$\delta_m^{\lambda_m}(k, \alpha) = \Delta_m^{AB}(\alpha) + \theta_{\lambda_m}, \quad (33)$$

where

$$\Delta_m^{AB}(\alpha) = \frac{\pi}{2}(|m| - |m + \alpha|), \quad (34)$$

is the usual phase shift of the AB scattering and

$$\theta_{\lambda_m} = \arctan(\mu_m^{\lambda_m}). \quad (35)$$

Therefore, the scattering operator $S_{\alpha, m}^{\lambda_m}$ (S -matrix) for the self-adjoint extension is

$$S_{\alpha, m}^{\lambda_m} = e^{2i\delta_m^{\lambda_m}(k, \alpha)} = e^{2i\Delta_m^{AB}(\alpha)} \left[\frac{1 + i\mu_m^{\lambda_m}}{1 - i\mu_m^{\lambda_m}} \right]. \quad (36)$$

Using Eq. (29), we have

$$S_{\alpha, m}^{\lambda_m} = e^{2i\Delta_m^{AB}(\alpha)} \times \left[\frac{B_k + i\lambda_m k^{2|m+\alpha|} \Gamma(1 - |m + \alpha|) \sin(\pi|m + \alpha|)}{B_k - i\lambda_m k^{2|m+\alpha|} \Gamma(1 - |m + \alpha|) \sin(\pi|m + \alpha|)} \right]. \quad (37)$$

Hence, for any value of the self-adjoint extension parameter λ_m , there is an additional scattering. If $\lambda_m = 0$, we achieve the corresponding result for the usual AB problem with Dirichlet boundary condition; in this case, we recover the expression for the scattering matrix found in Ref. [43], $S_{\alpha, m}^{\lambda_m} = e^{2i\Delta_m^{AB}(\alpha)}$. If we make $\lambda_m = \infty$, we get $S_{\alpha, m}^{\lambda_m} = e^{2i\Delta_m^{AB}(\alpha) + 2i\pi|m + \alpha|}$.

In accordance with the general theory of scattering, the poles of the S -matrix in the upper half of the complex plane [44] determine the positions of the bound states in the energy scale. These poles occur in the denominator of (37) with the replacement $k \rightarrow i\kappa$,

$$B_{i\kappa} + i\lambda_m (i\kappa)^{2|m+\alpha|} \Gamma(1 - |m + \alpha|) \sin(\pi|m + \alpha|) = 0. \quad (38)$$

Solving the above equation for E , we found the bound state energy

$$E = -\frac{2}{M} \left[-\frac{1}{\lambda_m} \frac{\Gamma(1 + |m + \alpha|)}{\Gamma(1 - |m + \alpha|)} \right]^{1/|m + \alpha|}, \quad (39)$$

for $\lambda_m < 0$. Hence, the poles of the scattering matrix only occur for negative values of the self-adjoint extension parameter. In this latter case, the scattering operator can be expressed in terms of the bound state energy

$$S_{\alpha,m}^{\lambda_m} = e^{2i\Delta_m^{AB}(\alpha)} \left[\frac{e^{2i\pi|m+\alpha|} - (\kappa/k)^{2|m+\alpha|}}{1 - (\kappa/k)^{2|m+\alpha|}} \right]. \quad (40)$$

The scattering amplitude $f_\alpha(k, \varphi)$ can be obtained using the standard methods of scattering theory, namely

$$\begin{aligned} f_\alpha(k, \varphi) &= \frac{1}{\sqrt{2\pi ik}} \sum_{m=-\infty}^{\infty} \left(e^{2i\delta_m^{\lambda_m}(k, \alpha)} - 1 \right) e^{im\varphi} \\ &= \frac{1}{\sqrt{2\pi ik}} \sum_{m=-\infty}^{\infty} \left(e^{2i\Delta_m(\alpha)} \left[\frac{1 + i\mu_m^{\lambda_m}}{1 - i\mu_m^{\lambda_m}} \right] - 1 \right) e^{im\varphi}. \end{aligned} \quad (41)$$

In the above equation we can see that the scattering amplitude differ from the usual AB scattering amplitude off a thin solenoid because it is energy dependent (cf., Eq. (29)). The only length scale in the nonrelativistic problem is set by $1/k$, so it follows that the scattering amplitude would be a function of the angle alone, multiplied by $1/k$ [45]. This statement is the manifestation of the helicity conservation [46]. So, one would expect the commutator of the Hamiltonian with the helicity operator, $\hat{h} = \boldsymbol{\Sigma} \cdot \boldsymbol{\Pi}$, to be zero. However, when calculated, one finds that

$$[\hat{H}, \hat{h}] = e\varepsilon \begin{pmatrix} 0 & (\boldsymbol{\sigma} \cdot \mathbf{B})(\boldsymbol{\sigma} \cdot \boldsymbol{\Pi}) \\ (\boldsymbol{\sigma} \cdot \mathbf{B})(\boldsymbol{\sigma} \cdot \boldsymbol{\Pi}) & 0 \end{pmatrix}, \quad (42)$$

which is nonzero for $\varepsilon \neq 0$. So, the inevitable failure of helicity conservation expressed in Eq. (41) follow directly from the deformation parameter ε and it must be related with the self-adjoint extension parameter, because the scattering amplitude depend on λ_m . Indeed, as it was shown in [20] it is possible to find a relation between the self-adjoint extension parameter and the coupling constant η in (14). By direct inspection we can claim that such relation is

$$\frac{1}{\lambda_m} = -\frac{1}{r_0^{2|m+\alpha|}} \left(\frac{\eta + |m+\alpha|}{\eta - |m+\alpha|} \right), \quad (43)$$

where r_0 is a very small radius smaller than the Compton wave length λ_C of the electron [47], which comes from the regularization of the δ function (for detailed analysis see [48]). The above relation is only valid for $\lambda_m < 0$ (when we have scattering and bound states), consequently we have $|\eta| \geq |m+\alpha|$ and due to $|m+\alpha| < 1$ it is sufficient to consider $|\eta| \geq 1$ to guarantee λ_m to be negative. A necessary condition for a δ function generates an attractive potential, which is able to support bound states, is that the coupling constant must be negative. Thus, the existence of bound states requires

$$\eta \leq -1. \quad (44)$$

Also, it seems from the above equation and from (14) that we must have $\alpha s < 0$ and there is a minimum value for the magnetic flux α . It is worthwhile observe that bound states and additional scattering still remain inclusive when $\varepsilon = 0$, i.e., no quantum deformation case, because the condition $\lambda_m < 0$ is satisfied, as it is evident from (43). It was shown in Refs. [20, 49].

Now, let us comeback to helicity conservation problem. In fact, the failure of helicity conservation expressed in Eq. (41), it stems from the fact that the δ function singularity make the Hamiltonian and the helicity non self-adjoint operators [50–53], hence their commutation must be analyzed carefully by considering first the correspondent self-adjoint extensions and after that compute the commutation relation, as we explain below. By expressing the helicity operator in terms of the variables used in (16), we attain

$$\hat{h} = \begin{pmatrix} 0 & -i \left(\partial_r + \frac{s|m+\alpha|+1}{r} \right) \\ -i \left(\partial_r - \frac{s|m+\alpha|}{r} \right) & 0 \end{pmatrix}. \quad (45)$$

This operator suffers from the same disease as the Hamiltonian operator in the interval $|m+\alpha| < 1$, i.e., it is not self-adjoint [54, 55]. Despite that on a finite interval $[0, L]$, \hat{h} is a self-adjoint operator with domain in the functions satisfying $\xi(L) = e^{i\theta}\xi(0)$, it does not admit a self-adjoint extension on the interval $[0, \infty)$ [56], and consequently it can be not conserved, thus the helicity conservation is broken due to the presence of the singularity at the origin [45, 51].

5. Conclusion

We have studied the AB problem within the framework of κ -deformed Schrödinger-Pauli equation. The new contribution to the Pauli's term is used to impose a upper bound in the deformation parameter, $\varepsilon < 2.27 \times 10^{-9} \text{ (eV)}^{-1}$. It has been shown that there is an additional scattering for any value of the self-adjoint extension parameter and for negative values there is non-zero energy bound states. On the other hand, the scattering amplitude show a energy dependency, it stems from the fact that the helicity operator and the Hamiltonian do not to commute. These results could be compared with those obtained in Ref. [49] where a relation between the self-adjoint extension parameter and the gyromagnetic ratio g was obtained. The usual Schrödinger-Pauli equation with $g = 2$ is supersymmetric [57] and consequently it admits zero energy bound states [58]. However, in the κ -deformed Schrödinger-Pauli equation $g \neq 2$ and supersymmetry is broken, giving rise to non-zero energy bound states. Changes in the helicity in a magnetic field represent a measure of the departure of the gyromagnetic ratio of the electron or muon from the Dirac value of $2e/2M$ [46]. Hence, the helicity nonconservation is related to nonvanishing value of $g - 2$.

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References

- [1] J. Lukierski, H. Ruegg, A. Nowicki, V. N. Tolstoy, Phys. Lett. B 264 (1991) 331.
- [2] J. Lukierski, A. Nowicki, H. Ruegg, Phys. Lett. B 293 (1992) 344.
- [3] A. Nowicki, E. Sorace, M. Tarlini, Phys. Lett. B 302 (1993) 419.
- [4] L. Biedenharn, B. Mueller, M. Tarlini, Phys. Lett. B 318 (1993) 613.
- [5] J. Lukierski, H. Ruegg, Phys. Lett. B 329 (1994) 189.
- [6] S. Majid, H. Ruegg, Phys. Lett. B 334 (1994) 348.
- [7] J. Lukierski, H. Ruegg, W. Zakrzewski, Ann. Phys. 243 (1995) 90.
- [8] A. Agostini, G. Amelino-Camelia, M. Arzano, Class. Quantum Grav. 21 (2004) 2179.
- [9] R. Aloisio, A. Galante, A. F. Grillo, F. Mndez, J. M. Carmona, J. L. Corts, J. High Energy Phys. 2004 (2004) 028.
- [10] D. Kovacević, S. Meljanac, A. Pachol, R. Strajn, Phys. Lett. B 711 (2012) 122.
- [11] M. Arzano, A. Marcianò, Phys. Rev. D 76 (2007) 125005.
- [12] P. Roy, R. Roychoudhury, Phys. Lett. B 339 (1994) 87.
- [13] P. Roy, R. Roychoudhury, Phys. Lett. B 359 (1995) 339.
- [14] J. Ndimubandi, Europhys. Lett. 39 (1997) 583.
- [15] P. Roy, R. Roychoudhury, Mod. Phys. Lett. A 10 (1995) 1969.
- [16] Y. Aharonov, D. Bohm, Phys. Rev. 115 (1959) 485.
- [17] A. Das, H. Falomir, M. Nieto, J. Gamboa, F. Méndez, Phys. Rev. D 84 (2011) 045002.
- [18] M. A. Hohensee, B. Estey, P. Hamilton, A. Zeilinger, H. Müller, Phys. Rev. Lett. 108 (2012) 230404.
- [19] F. Correa, H. Falomir, V. Jakubsky, M. S. Plyushchay, J. Phys. A 43 (2010) 075202.

- [20] F. M. Andrade, E. O. Silva, M. Pereira, Phys. Rev. D 85 (2012) 041701(R).
- [21] L. Vaidman, Phys. Rev. A 86 (2012) 040101.
- [22] K. Fang, Z. Yu, S. Fan, Phys. Rev. Lett. 108 (2012) 153901.
- [23] H. Belich, E. O. Silva, J. Ferreira, M. M., M. T. D. Orlando, Phys. Rev. D 83 (2011) 125025.
- [24] M. A. Anacleto, F. A. Brito, E. Passos, Phys. Rev. D 86 (2012) 125015.
- [25] B. S. Kay, U. M. Studer, Commun. Math. Phys. 139 (1991) 103.
- [26] W. Bulla, F. Gesztesy, J. Math. Phys. 26 (1985) 2520.
- [27] S. Albeverio, F. Gesztesy, R. Hoegh-Krohn, H. Holden, Solvable Models in Quantum Mechanics, AMS Chelsea Publishing, Providence, RI, second edition, 2004.
- [28] H. J. de Vega, Phys. Rev. D 18 (1978) 2932.
- [29] R. H. Brandenberger, A.-C. Davis, A. M. Matheson, Nucl. Phys. B 307 (1988) 909.
- [30] M. G. Alford, F. Wilczek, Phys. Rev. Lett. 62 (1989) 1071.
- [31] E. Harikumar, M. Sivakumar, N. Srinivas, Mod. Phys. Lett. A 26 (2011) 1103.
- [32] P. J. Mohr, B. N. Taylor, D. B. Newell, Rev. Mod. Phys. 84 (2012) 1527.
- [33] G. Gabrielse, D. Hanneke, T. Kinoshita, M. Nio, B. Odom, Phys. Rev. Lett. 97 (2006) 030802.
- [34] F. Gesztesy, S. Albeverio, R. Hoegh-Krohn, H. Holden, J. Reine Angew. Math. 1987 (1987) 87.
- [35] L. Dabrowski, P. Stovicek, J. Math. Phys. 39 (1998) 47.
- [36] R. Adami, A. Teta, Lett. Math. Phys. 43 (1998) 43.
- [37] M. Reed, B. Simon, Methods of Modern Mathematical Physics. II. Fourier Analysis, Self-Adjointness., Academic Press, New York - London, 1975.
- [38] J. J. Sakurai, J. Napolitano, Modern Quantum Mechanics, Addison-Wesley, 2nd ed. edition, 2011.
- [39] J. Audretsch, U. Jasper, V. D. Skarzhinsky, J. Phys. A 28 (1995) 2359.
- [40] F. A. B. Coutinho, Y. Nogami, J. Fernando Perez, Phys. Rev. A 46 (1992) 6052.
- [41] C. R. de Oliveira, M. Pereira, J. Phys. A 43 (2010) 354011.

- [42] M. Abramowitz, I. A. Stegun (Eds.), Handbook of Mathematical Functions, New York: Dover Publications, 1972.
- [43] S. N. M. Ruijsenaars, Ann. Phys. (NY) 146 (1983) 1.
- [44] K. Bennaceur, J. Dobaczewski, M. Ploszajczak, Phys. Rev. C 60 (1999) 034308.
- [45] A. S. Goldhaber, Phys. Rev. D 16 (1977) 1815.
- [46] J. J. Sakurai, Advanced Quantum Mechanics, Addison Wesley, 1967.
- [47] M. Bordag, S. Voropaev, Phys. Lett. B 333 (1994) 238.
- [48] F. M. Andrade, E. O. Silva, M. Pereira, arXiv:quant-ph/1207.0214 (2012).
- [49] M. Bordag, S. Voropaev, J. Phys. A 26 (1993) 7637.
- [50] Y. Kazama, C. N. Yang, A. S. Goldhaber, Phys. Rev. D 15 (1977) 2287.
- [51] B. Grossman, Phys. Rev. Lett. 50 (1983) 464.
- [52] N. Ganoulis, Phys. Lett. B 298 (1993) 63.
- [53] A. Davis, A. Martin, N. Ganoulis, Nucl. Phys. B 419 (1994) 323.
- [54] F. A. B. Coutinho, J. F. Perez, Phys. Rev. D 49 (1994) 2092.
- [55] V. S. Araujo, F. A. B. Coutinho, J. F. Perez, J. Phys. A 34 (2001) 8859.
- [56] G. Bonneau, J. Faraut, G. Valent, Am. J. Phys. 69 (2001) 322.
- [57] R. Jackiw, Phys. Rev. D 29 (1984) 2375.
- [58] Y. Aharonov, A. Casher, Phys. Rev. A 19 (1979) 2461.